Abstract—One of the primary techniques for target tracking is the Kalman filter or one of its many variants. While the Kalman filter has been the staple of the tracking community, it has been shown to have drawbacks. When a target performs a maneuver, the tracking solution of the filter can experience deleterious issues. The most common issue is a lag in the position of the target track compared to the true target position as the target performs its maneuver. A more problematic issue occurs when the filter covariance collapses which requires the filter to be reinitialized. While techniques exist that compensate for maneuvers, they rely on detecting the error in the estimated trajectory and the measured target position to generate their response. In this effort, a maneuver detection routine is developed that can be used in conjunction with the standard maneuver compensation approaches. This routine validates the existence of a maneuver more quickly than using inherent detection methods of the other methods. The maneuver detection is performed by an evidence accrual system that uses a fuzzy Kalman filter to incorporate new information and provide a level of evidence that maneuver is occurring. The input data uses behavior characteristics of the Kalman gain vector from the tracking algorithm.

Keywords—Kalman gain, monitoring, pattern detection, fuzzy logic, evidence accrual, maneuver detection

I. INTRODUCTION

The predominant kinematic target-tracking algorithm is the Kalman filter or one of its variants [1]. A problem with these approaches is that, if the target dynamics, such as a target maneuver, are not properly modeled, poor tracking behavior can ensue. Fig. 1 is the reconstruction from [2] of the comparison of truth against a target track constructed a poorly-modeled target with a ballistic trajectory. The tracker is an extended Kalman filter (EKF). The target track continues to elevate even after the target reaches apogee. The track then slowly tries to compensate for its offset over the rest of the trajectory. This comparative-result exemplifies that target-tracking can experience deleterious effects when a maneuver occurs. In this case, a position lag in the target track occurs. Tracking problems become more prevalent when the measurement lies the unobservable space of the track kinematics as seen in [3]. In those cases, where a bearing-only tracker is used, the filter can become numerically unsound and require a re-initialization.

To reduce the deleterious effects that maneuvers have on a tracking system, compensation approaches have been developed. The most widely used technique is the interacting multiple model (IMM) and its variants [4]. The IMM uses different maneuver models and compares the residuals of each measurement to determine which model best fits the current behavior. By interpolating between the models based on the residual score, a more accurate motion model is created. Other techniques include adaptive Kalman filters such as a neural extended Kalman Filter (NEKF) [2,5]. These methods use the residual to adapt their maneuver parameters to more closely model that of the actual target behavior. All of these techniques depend on the residual to determine if a maneuver is occurring. While the residual is an effective measure, variations in uncertainty can mask the maneuver until it gets significantly large. Fortunately, there exist other metrics as part of the Kalman filter that can detect a maneuver.

Recently, in [6], analysis showed that the Kalman gain component of the Kalman filter exhibits various behaviors if the tracked target is performing a maneuver. While an individual Kalman gain behavior may not alone indicate a target is maneuvering, the monitoring of multiple behaviors together can increase the probability of properly and quickly detecting a maneuver. Through the development of a maneuver detector, it is possible to coordinate with the compensation algorithms to reduce their reaction times.

To fuse the Kalman gain behaviors together and estimate if a maneuver is occurring, an evidence accrual technique, referred to as feature object extraction (FOX), is employed. The technique was first developed as a target-classification approach [7]. It utilizes a tree structure to develop relationships between lower levels of evidence and the fused outputs. The injection of data is performed using a fuzzy Kalman filter [8] that injects both fuzzy measurements and measurement uncertainty into the system.

To overview the development of this approach and analyze its capability, the paper is decomposed into five sections. Section 2 overviews the EKF, which is the most prominent variant of the Kalman filter used in tracking. Section 3
summarizes the Kalman gain behaviors that indicate when a target is maneuvering. Section 4 describes the FOX evidence accrual system and how its implemented for maneuver detection. In Section 5 a maneuver tracking problem is described and the detection capabilities of the FOX approach using Kalman gains are provided.

II. THE EXTENDED KALMAN FILTER FOR TRACKING

Since the measurements from sensors usually provide a nonlinear relationship between the state representation of the target track

\[ \mathbf{x}^{T} = \begin{bmatrix} x & \dot{x} & y & \dot{y} & z & \dot{z} \end{bmatrix} \] (1)

and the measurement space, the extended Kalman filter (EKF) is the preferred tracking algorithm. Using active sensors, such as radar, a complete measurement space for a three-dimensional target-track would be a range/bearing/elevation report

\[ \mathbf{h}(\mathbf{x}) = \begin{bmatrix} \text{range} \\ \text{bearing} \\ \text{elevation} \end{bmatrix} = \begin{bmatrix} \rho \\ \beta \\ \epsilon \end{bmatrix} = \begin{bmatrix} \sqrt{(x_{tgt} - x_{plt})^2 + (y_{tgt} - y_{plt})^2 + (z_{tgt} - z_{plt})^2} \\ \arctan\left(\frac{x_{tgt} - x_{plt}}{y_{tgt} - y_{plt}}\right) \\ \arctan\left(\frac{z_{tgt} - z_{plt}}{\sqrt{(x_{tgt} - x_{plt})^2 + (y_{tgt} - y_{plt})^2 + (z_{tgt} - z_{plt})^2}}\right) \end{bmatrix} \] (2)

where the subscript \( tgt \) denotes target component and the subscript \( plt \) denotes the platform.

These measurements are used as the inputs to the tracking algorithm. The EKF uses its estimate of the measurement and the residual between the estimated and reported measurements to correct its state-estimate of the target kinematics. The process of EKF to accomplish this is defined in (3a-e) as

\[ K_k = P_{k|k-1}H_k^T (H_k P_{k|k-1}H_k^T + R_k)^{-1} \] (3a)

\[ x_{k|k} = x_{k|k-1} + K_k (z_k - \mathbf{h}(x_{k|k-1})) \] (3b)

\[ P_{k|k} = (I - K_k H_k) P_{k|k-1} \] (3c)

\[ \mathbf{x}_{k+1|k} = \mathbf{f}(\mathbf{x}_k) \] (3d)

\[ P_{k+1|k} = \mathbf{F}_k P_{k|k} \mathbf{F}_k + \mathbf{Q}_k \] (3e)

where \( \mathbf{H} \) is the Jacobian of the output-coupling function

\[ \mathbf{H} = \begin{bmatrix} \frac{\partial \rho}{\partial x} & 0 & \frac{\partial \rho}{\partial y} & 0 & \frac{\partial \rho}{\partial z} & 0 \\ \frac{\partial \beta}{\partial x} & 0 & \frac{\partial \beta}{\partial y} & 0 & 0 & 0 \\ \frac{\partial \epsilon}{\partial x} & 0 & \frac{\partial \epsilon}{\partial y} & 0 & \frac{\partial \epsilon}{\partial z} & 0 \end{bmatrix} \] (4)

The function \( \mathbf{f} \) is the modeled target dynamics, and the matrix \( \mathbf{F} \) is the associated Jacobian. Usually, the target dynamics are modeled as

\[ \mathbf{F} = \begin{bmatrix} \mathbf{F}_{2x2} & \mathbf{0}_{2x2} & \mathbf{0}_{2x2} \\ \mathbf{0}_{2x2} & \mathbf{F}_{2x2} & \mathbf{0}_{2x2} \\ \mathbf{0}_{2x2} & \mathbf{0}_{2x2} & \mathbf{F}_{2x2} \end{bmatrix} \] (5a)

where

\[ \mathbf{F}_{2x2} = \begin{bmatrix} 1 & dt \\ 0 & 1 \end{bmatrix} \] (5b)

The process noise \( \mathbf{Q} \), which indicates the accuracy of the system dynamics, is usually modeled as integrated white noise [4]. As the process noise is increased, the dynamic model \( \mathbf{F} \) in (3d-e) is weighted less. This results in the measurement becoming more dominant in the processing which allows more of the measurement noise to be passed thought the filter to the track solution. As the process noise is decreased, the reaction of the tracking algorithm becomes less responsive to the measurement and more smoothing takes place.

The Kalman gain of (3a) is affected by the process noise, the error covariance \( \mathbf{P} \), and the state estimate \( \mathbf{x} \). This is not explicit in the equation and is often overlooked. The state estimate is injected in to the Kalman gain though the output-coupling Jacobian \( \mathbf{H} \). The gain is also affected by the
measurement noise \( R \). The subscript \( k \) indicates discrete time, with \( k|k \) is the estimate at the time \( k \) given all the information up to that time and \( k+1|k \) is the estimate for time \( k+1 \), given all the information up through time \( k \).

III. MONITORING THE KALMAN GAIN

While it is well known that applications monitor the behavior of the error covariance, (3c) and (3e), to determine how the filter is behaving, the Kalman gain \( K \) contains more information. In the EKF, for example, the Kalman gain contains both information about the error covariance and the state vector.

In [6], the behaviors of the Kalman vector components were analyzed as they pertained to target maneuvers. The Kalman gains for several two-dimensional target-tracking problems were generated to determine how the Kalman gains are affected by target maneuvers. An example of the Kalman gain’s capability to detect maneuvering targets is that of a sensor platform with constant velocity while tracking a target that maneuvers part way through the scenario. The example is shown in Fig. 2. This represents a concept in a submarine-tracking problem where a target is tracked passively and maneuvers to clear baffles to detect a trailing ship. The surface ship’s passive tracking system cannot easily track a target through a maneuver. The filter may even go numerically unsound. Fig. 3 shows the behavior of the four Kalman gains at the time of the maneuver. The Kalman gains show different behaviors, which include a high frequency behavior, a sharp transient, a significant change in the slope of the gain, and a zero-crossing. This example, of all those in [6], showed several simultaneous behaviors that occur to the Kalman gains during the maneuver. Other examples showed similar behaviors but also noted that platform’s behavior also had an effect on the zero-crossing behavior of the gains and transient chirps. Fortunately, these can be monitored from known platform behavior. The maneuvers of the platform have been noted to cause smooth but significant changes in the slope of the gains, which return to similar to gain values after the maneuver that existed before the maneuver. Another event in the Kalman gain is continual growth. In the most egregious cases of maneuver effects, the Kalman gain after a maneuver is unable to settle. In such cases, the EKF becomes unstable in its behavior. However, it is the gain, which precedes the covariance and state failures.

Thus, by monitoring the frequency behavior, the zero-crossings, the transient behavior, the slope performance, and the ownship (platform) maneuvers, the target’s maneuvering can be determined.

IV. EVIDENCE ACCRUAL USING FEATURE OBJECT EXTRACTION

While these individual Kalman-gain behavior measures can indicate a maneuver is occurring, their accuracy is not perfect. To utilize these behaviors to their fullest potential, an evidence accrual technique is proposed to combine the information and provide a level of evidence that a maneuver is occurring or ceased to be. While techniques such as Dempster-Shafer [9,10] or Bayesian taxonomy [11] could be of use, the aforementioned FOX approach is proposed. This fuzzy-based evidence accrual technique has many advantages including a measure of uncertainty with the level of reported evidence. The incorporation of the measure of target-maneuver evidence along with its associated quality provides detection with a level of confidence in the detection. Thus, the report of a detected maneuver, which can be the beginning of a turn or the ending of a turn, to the maneuver-tracking algorithms can be used to provide an early response to a maneuver, increase confidence in the maneuver so that more adjustment can be made, or identify possible false positive maneuver detections.

The FOX technique decomposes a complex classification problem into a series of smaller and simpler problems. Fig. 4 shows a tree representation that is used to generate the decomposition of the maneuver-detection evidence-accrual problem.

![Fig. 2. Exemplar of a non-maneuvering platform tracking maneuvering target](image1)

![Fig. 3. The behavior of the Kalman gains during a target maneuver](image2)
For maneuver detection, the top level is Detected Maneuver. The next level is comprised of individual-event detections of the Kalman gain: Zero Crossing, Gain Frequency, and Slope Variation. These three components are combined to create the Detected Maneuver score. As the evidence levels in these states increase from 0 to 1, the overall detection level varies based not only on the lower level scores but the quality level as well. Poor quality scores are weighted less than higher quality scores. Another important fact about the FOX approach is that unlike the states of Markov chain [12], these states need not be disjoint nor do the states need be a complete representation of all the states of the system. In Fig. 4, the shaded states indicate low-level input nodes. These represent the elemental measurements such as the measured frequencies or number of zero crossings. The unshaded nodes are referred to as states of interest. These are states of combined information.

The level of evidence of a state of interest can be generated in two ways. The first is through direct observation, which implies a direct measurement is available as shown with the shaded nodes. The evidence is then processed through a substate, which are depicted as a clear node. The tree of Fig. 4 shows some substates have multiple injection nodes flowing into them. For the maneuver detection application, the injected evidence flows through a fuzzy Kalman filter that takes the multiple inputs and creates a single output fuzzy measure. Substates that do not have direct evidence injection are generated using systems theory. The substates connect to state of interest through links (the solid lines) that represent the functional relationship. Direct observations of states feed the state of interest or substate through a state estimation process.

A state of interest and its direct-substates are represented in vector form as

\[ \mathbf{x}_k = [x_{k1}, x_1, x_2, \ldots, x_n] \quad (6) \]

Using first-order observer decomposition, the evidence dynamics are given as

\[ \mathbf{x}_{k+1} = \begin{bmatrix} f(x_{a}, x_{a1}, \ldots, x_{am}) \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \quad P_{k+1} = \begin{bmatrix} f_{a} P_{a}^{-1} f_{a}^{T} + f_{a} P_{a}^{-1} f_{d}^{T} + \cdots + f_{a} P_{a}^{-1} f_{n}^{T} + q_{k} \end{bmatrix} \quad (7) \]

The associated error covariance or uncertainty equation is represented by (8).

\[ P_k = f_{a} P_{a}^{-1} f_{a}^{T} + f_{a} P_{a}^{-1} f_{d}^{T} + \cdots + f_{a} P_{a}^{-1} f_{n}^{T} + q_{k} \quad (8) \]

This decomposition of the problem into the individual substates reduces the complex model of the interactions of information into simplified operations.

The estimation routine which is used to inject information is the fuzzy Kalman filter (FKF) developed by Watkins [8] and modified in [7]. The implementation is a straightforward variant of the standard Kalman filter with a modification in the update equations. The equations for the FKF are given as

\[ K = P_{a}^{-1} H^{T} (H P_{a}^{-1} H^{T} + \text{mom}_{1}(m_{ad}))^{-1} \quad (9a) \]

\[ x_{k|k} = x_{k|k-1} + K(x_{m} - H x_{k|k-1}) \quad (9b) \]

\[ P_{k|k} = (I - KH) P_{k|k-1} \quad (9c) \]

\[ x_{k+1|k} = F x_{k|k} \quad (9d) \]

\[ P_{k+1|k} = F P_{k|k} F^{T} + Q_{k} \quad (9e) \]

Comparing (3) and (9), only the Kalman gain equation, (9a), and the state update equation, (9b) are different than their counterparts in (3). The fuzzy measures are incorporated by using the first moment, indicated as \text{mom}_{1}, of the consequent fuzzy membership function, referred to as \text{mom}_{1}, or the membership adjunct.

If the measurement and uncertainty values are crisp, the FKF devolves into the standard Kalman filter. The use of fuzzy logic provides for simplified linguistic based conversion from the measurement coordinate systems to the evidence space, which has been defined as a value between 0 and 1.

The determination whether the fuzzy measure is used or a crisp measure is based on the data. A true measure such as the number of zero crossings or a known even as the knowledge of a platform maneuver would be a crisp value. However, the mapping of data such to a determination such as the frequency behavior or degree of slope change that map into groupings such as high frequency changes and low frequency changes would be fuzzy.
To generate the injection evidence, the following antecedent functions were considered:

- Frequency amplitude variation (Fuzzy)
- Number of zero crossings within frequency band (Crisp) and existence of a platform maneuver (Crisp)
- Number of slope changes (Crisp), average size of slope changes (Fuzzy), and existence a platform maneuver (Crisp)

The function relating the three sets of substate evidence to the Detected Maneuver state is defined the limited linear combination,

\[ x_{MD}(k) = \min(1, 0.7x_{MDfreq}(k) + 0.7x_{MDzc}(k) + 0.7x_{MDslope}(k)) \]  \hspace{1cm} \text{(10)}

where \( x_{MD}(k) \) is the estimated level of evidence at time \( k \) for detection of a maneuver state. The states that comprise \( x_{MD} \) are \( x_{MDfreq} \), the frequency behavior of the Kalman gain used to detect a maneuver, \( x_{MDzc} \) is the incidence of zero-crossings for a Kalman gain value, and \( x_{MDslope} \) is the change in slope of the Kalman gain value that indicates a maneuver. In this case, the score was not directly weighted by the measurement uncertainty.

The subnode \( x_{MDfreq} \) is defined as

\[ x_{MDfreq}(k) = 0.9x_{MDfreq}(k-1) + 0.2x_{K11}(k) + 0.3x_{K31}(k) + 0.3x_{K12}(k) + 0.2x_{K22}(k) + 0.3x_{K32}(k) + 0.2x_{K42}(k), \]  \hspace{1cm} \text{(11)}

where the subscript \( Kij \) indicates the \( i \), \( j \)th element of the Kalman gain vector. The zero crossings is a direct observation feed based on the count that is binned as crisp ranges of values.

The subnode \( x_{MDslope} \) is defined as

\[ x_{MDslope}(k) = 0.9x_{MDslope}(k-1) + 0.2x_{K12}(k) + 0.3x_{K32}(k), \]  \hspace{1cm} \text{(12)}

V. EXAMPLES AND ANALYSIS

For an example case, a target progressing through multiple maneuvers was generated. The problem represents a two-dimensional target-tracking problem.

Fig. 5 depicts the trajectory of the target and the platform. The scenario is 1000 s long. The target starts as a stationary target for the first 100 s. It then accelerates to 10 m/s at 0° from the x-axis for 20 s. At time 170 s into the scenario, the target maintains its speed but changes heading for 30 s until it reaches a heading of 20° from the x-axis. The target changes its heading to -15° at the same speed at time 399 s into the scenario. The maneuver takes place over 40 s. The next maneuver occurs 899 s into the scenario. The target changes its heading to -60° from the x-axis over 25 s.

For the last maneuver, the target increases its speed at the same heading to 20 m/s over 10 s. The target is assumed to start at (0,0) in the scenario map. The platform initially starts at (-500 m, -1000 m) on the scenario map. The platform initially moves at 8 m/s with a heading of -30°. At 449 s into the scenario, the platform changes heading to 12° and a speed of 12 m/s over 60 s. The sensor on the platform is defined to provide a range-bearing report at 1s updates. The accuracy of the sensor is ±1 m in the range and 1.5° in bearing.

The resulting Kalman gains for the four states are depicted in Figs. 6 and 7. Fig. 6 denotes the gains associated with the range measurements (Column1 of the gain matrix \( K \)). Fig. 7 are the gains over the scenario associated with the bearing measurement.
The gains are in order of x-position, x-velocity, y-position, and y-velocity for each column. The events of the target maneuvers and the single platform maneuver are denoted by the vertical lines, which can then be compared to the Kalman gain variations.

The slope scores were generated first by smoothing the gains. The slopes were calculated by taking points five samples apart. The resulting slopes were smoothed over time using a weighted smoothing algorithm. A score was generated between 0 and 1, inclusive, by comparing the current slope to the slope 10 samples prior.

Two fuzzy logic inputs were used. Each contained seven antecedent membership functions. The first three of these indicate large, medium, and small negative slopes, using trapezoidal membership functions. There was one trapezoidal membership function around 0. The three remaining membership functions indicate positive slopes. The inference engine maps similar slopes to a good quality score and highly varying slopes to a low-quality score. The consequent membership functions are four triangular membership functions, spaced evenly from 0 to 1.

The two patterns that demonstrated detection capabilities are the slopes of the Kalman gains and a windowed FFT of the gains. The variation in the slopes of the bearing-related gains (the second column of the Kalman gain) and the velocity components of the range elements of Kalman gain (the second and fourth elements of the first column of the gain-vector), all indicated very good, good, or fair performance in detecting the maneuvers. For the FFT, only the position elements of the bearing measurements showed detection capabilities both of which were rated good. The ranking of the quality was used to scale the uncertainty used in the Kalman filter to inject the evidence to the maneuver detection evidence accrual algorithm.

Each gain and detection approach generated its own score. The scores were incorporated into base nodes of the evidence tree. The zero crossing events did not meet the criteria. While crossings occurred, they mimicked noise or graceful transitions other than a burst or sudden change and thus did not trigger a score.

The Kalman gains clearly indicate that the bearing-based gains provide the most information to the detection of a maneuver. This is most likely a result of the fact that the maneuvers were most clearly evident in cross-range space.

The maneuver detection algorithm without and with eliminating the sensor-platform’s maneuver are shown in Fig. 8 and 9, respectively. Fig. 8 shows a detection of the first maneuver and ownship maneuver if a high threshold is used. If the threshold were reduced to 0.35 all of the maneuvers would be detected. The close proximity of the maneuvers show that the tracking system does not quickly reset, and the closely spaced maneuvers appears as one long maneuver. The ownship maneuver is a long maneuver and has a large effect on the tracker’s performance. The results of Fig. 8 clearly indicate that without removing the knowledge of the platform maneuver a target maneuver would be declared even though no such maneuver was occurring.
Fig. 9 depicts the results when sensor platform has its maneuver removed from the scoring. As long as the platform detection was flagged, the score was set to zero. However, the platform maneuver does have a long-term effect on the maneuver detection score. The settling of the score could mask real target maneuvers. However, the change in the Kalman gain residual would alert the techniques such as the IMM and NEKF that a model change should be employed when it should not. The FOX model could be changed or made adaptive to correct for the platform maneuver impacts.

VI. CONCLUSIONS

In this paper, the FOX evidence accrual technique was applied to the problem of detecting target maneuvers using measures from the Kalman gain behaviors from an EKF tracking routine. This maneuver detection algorithm can detect target maneuvers.

The basic concept demonstrated that the significant element is the pattern recognition tuning to create the scores that feed the system. These used simple fuzzy scoring algorithms that used the measures of the slope and frequency measures Crisp values of platform maneuvers, zero-crossings, and the number of slope changes were also incorporated. The uncertainties of the scores were based the quality observed by empirical analysis. The uncertainties were only used for the examples as part of the FKF inputs of the data injection.

While this initial analysis indicates that the evidence accrual system is able to detect the maneuvers of the target, it revealed that more analysis and development is necessary. The scoring approaches need to be improved to provide higher scores for maneuvers without false alarms becoming a issue. The issue with platform maneuvers masking actual target maneuvers needs to be addressed. This may include a forgetting factor to lower the impact of specific Kalman gain behaviors. Also, the use of the quality scores in the evidence propagation and their mapping is necessary next step in this programs development. These steps are required before incorporating the FOX maneuver detector into a system to aid maneuver compensating tracking algorithms.

REFERENCES